

**OLIMPIADA SATELOR CLUJENE**  
**MATEMATICĂ- ETAPA JUDEȚEANĂ**  
**10.03.2023**

**Barem de corectare. Clasa a VIII-a**

**Problema 1 (7 puncte)**

a)  $a^2 + 1 \geq 2a \Leftrightarrow a^2 - 2a + 1 \geq 0 \Leftrightarrow (a - 1)^2 \geq 0$ ..... (2p)

b)  $x^2 + \frac{1}{x^2} \geq 2$  ..... (1p)

$\Rightarrow x^2 + \frac{1}{x^2} + 7 \geq 9 \Rightarrow \sqrt{x^2 + \frac{1}{x^2} + 7} \geq 3$  ..... (1p)

$\sqrt{9x^2 + 4y^2 - 12xy + 4} = \sqrt{(3x - 2y)^2 + 4} \Rightarrow \sqrt{9x^2 + 4y^2 - 12xy + 4} \geq 2$  ..... (1p)

$\sqrt{x^2 + \frac{1}{x^2} + 7} + \sqrt{9x^2 + 4y^2 - 12xy + 4} \geq 5$ ..... (1p)

Inegalitațiile de mai sus sunt egalități dacă  $x^2 = 1$  și  $3x - 2y = 0 \Rightarrow x = 1 \Rightarrow y = \frac{3}{2}$

și  $x = -1 \Rightarrow y = -\frac{3}{2}$  ..... (1p)

**Problema 2 (7 puncte)**

a)  $x^3 - 3x^2 - x + 3 = (x - 3)(x^2 - 1) = (x - 3)(x - 1)(x + 1)$  ..... (2p)

b)  $E(x) = \frac{(x-3)(x-1)(x+1)}{3(x-4)(x+4)} : \left( \frac{x^2-x-6}{3x(x-2)(x+2)} \cdot \frac{x(x-2)(x-1)}{2(x-4)} \right)$ ..... (4p)

$E(x) = \frac{2(x+1)}{x+4}$  ..... (1p)

**Problema 3 (7 puncte)**

Desen corect..... (1p)

a)  $AB = 10 \text{ cm}$   
 $A_{ABCD} = 100 \text{ cm}^2$  ..... (2p)

b)  $\left. \begin{array}{l} BO \perp (VAC) \\ \text{fie } OE \perp VC \end{array} \right\} \Rightarrow VC \perp (OBE) \Rightarrow VC \perp BE \Rightarrow BE \perp VC$  ..... (1p)

$\left. \begin{array}{l} (VAC) \cap (VBC) = VC \\ OE \perp VC, OE \subset (VAC) \\ BE \perp VC, BE \subset (VBC) \end{array} \right\} \Rightarrow (\sphericalangle(VAC), (VBC)) = \sphericalangle(OEB)$ ..... (1p)

$tg \sphericalangle(OEB) = \frac{OB}{OE} = \frac{5\sqrt{2}}{5} = \sqrt{2}$  ..... (1p)

c) Desfășurarea  
 $VP \perp DC \Rightarrow \Delta VPT \sim \Delta MCT$  ,  $CT = \frac{5(\sqrt{3}-1)}{2} \text{ cm}$  ..... (1p)

**Problema 4 (7 puncte)**

Desen corect ..... (1p)

a)  $AB = 8 \text{ cm}$  ..... (2p)

$\Rightarrow$  diagonala =  $8\sqrt{3} \text{ cm}$  ..... (1p)

b)  $\left. \begin{array}{l} B'D' \perp A'C' \\ B'D' \perp AA' \end{array} \right\} \Rightarrow B'D' \perp (AA'C) \Rightarrow B'D' \perp A'C$  ..... (1p)

$A'O' \parallel AC \Rightarrow \Delta A'MO' \sim \Delta CMA$  unde  $A'C \cap AO' = \{M\}, A'C' \cap D'B' = \{O'\}$   
 $\Rightarrow \frac{A'M}{MC} = \frac{MO'}{MA} = \frac{A'O'}{CA} = \frac{1}{2} \Rightarrow A'M = \frac{8\sqrt{3}}{3} \text{ cm}, O'M = \frac{4\sqrt{6}}{3} \text{ cm}, A'O' = 4\sqrt{2} \text{ cm}$   
 $\Rightarrow \Delta A'MO'$  dreptunghic în  $M \Rightarrow A'C \perp AO'$  ..... (1p)

$\left. \begin{array}{l} A'C \perp B'D' \\ A'C \perp AO' \end{array} \right\} \Rightarrow A'C \perp (AB'D')$ ..... (1p)